

Cosmic Constraint on Ricci Dark Energy Model

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In this paper, a holographic dark energy model, dubbed Ricci dark energy, is confronted with cosmological observational data from type Ia supernovae (SN Ia), baryon acoustic oscillations (BAO) and cosmic microwave background (CMB). By using maximum likelihood method, it is found out that Ricci dark energy model is a viable candidate of dark energy model with the best fit parameters: $\Omega_{m0} = 0.34 \pm 0.04$, $\alpha = 0.38 \pm 0.03$ with 1σ error. Here, α is a dimensionless parameter related with Ricci dark energy ρ_R and Ricci scalar R , i.e., $\rho_R \propto \alpha R$.

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The observation of the Supernovae of type Ia [1, 2] provides the evidence that the universe is undergoing accelerated expansion. Jointing the observations from Cosmic Background Radiation [3, 4] and SDSS [5, 6], one concludes that the universe at present is dominated by 70% exotic component, dubbed dark energy, which has negative pressure and pushes the universe to accelerated expansion. Of course, the accelerated expansion can attribute to the cosmological constant naturally. In fact, the mostly observational data is roughly consistent with the predictions of the Λ CDM model [7, 8, 9]. However, it suffers the so-called *fine tuning* and *cosmic coincidence* problem. To avoid these problems, dynamic dark energy models are considered, such as quintessence [10, 11, 12, 13, 14, 15], phantom [16], quintom [17] and holographic dark energy [18, 19] etc. The holographic dark energy is considered as a dynamic vacuum energy. It is constructed by considering the holographic principle and some features of quantum gravity theory. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and has relations with the area of its boundary. By applying the principle to cosmology, one can obtain the upper bound of the entropy contained in the universe. For a system with size L and UV cut-off Λ without decaying into a black hole, it is required that the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus $L^3 \rho_\Lambda \leq LM_{pl}^2$. The largest L allowed is the one saturating this inequality, thus $\rho_\Lambda = 3c^2 M_{pl}^2 L^{-2}$, where c is a numerical constant and M_{pl} is the reduced Planck Mass $M_{pl}^{-2} = 8\pi G$. It just means a *duality* between UV cut-off and IR cut-off. The UV cut-off is related to the vacuum energy, and IR cut-off is related to the large scale of the universe, for example Hubble horizon, event horizon or particle horizon as discussed by [18, 19]. In the paper [19], the author took the future event horizon

$$R_{eh}(a) = a \int_t^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{Ha'^2} \quad (1)$$

as the IR cut-off L . As pointed out in [19], it can reveal the dynamic nature of the vacuum energy and provide a solution to the *fine tuning* and *cosmic coincidence* problem. In this model, the value of parameter c determines the property of holographic dark energy. When $c \geq 1$, $c = 1$ and $c \leq 1$, the holographic dark energy behaviors like quintessence, cosmological constant and phantom respectively.

Recently, Gao, *et. al.* took the Ricci scalar as the IR cut-off and named it Ricci dark energy [20]. In that paper [20], it has shown that this model can avoid the causality problem and naturally solve the coincidence problem of dark energy. In this paper, we will revisit the Ricci dark energy model and use cosmic observations to constrain the model parameters. At first, we give a brief review of the Ricci dark energy (RDE). We consider a Friedmann-Robertson-Walker universe filled with cold dark matter and RDE. Its metric is written as

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \quad (2)$$

where $k = 1, 0, -1$ for closed, flat and open geometries respectively. The Friedmann equation is

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_R) - \frac{k}{a^2}, \quad (3)$$

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where H is the Hubble parameter, ρ_m and ρ_R denote the energy density of cold dark matter and Ricci dark energy respectively. As suggested by Gao *et. al.*, the RDE is proportional to the Ricci scalar

$$R = -6 \left(\dot{H} + 2H^2 + \frac{k}{a^2} \right). \quad (4)$$

Then, it is given as

$$\rho_R = \frac{3\alpha}{8\pi G} \left(\dot{H} + 2H^2 + \frac{k}{a^2} \right) \propto R, \quad (5)$$

where α is dimensionless model parameter which can be determined by confronting with cosmic observations. Now, the Friedmann equation (3) can be rewritten as

$$H^2 = \frac{8\pi G}{3} \rho_{m0} e^{-3x} + (\alpha - 1) k e^{-2x} + \alpha \left(\frac{1}{2} \frac{dH^2}{dx} + 2H^2 \right) \quad (6)$$

where $x = \ln a$. Dividing above equation by H_0^2 in both sides, one has

$$E^2 = (1 - \alpha) \Omega_k e^{-2x} + \Omega_{m0} e^{-3x} + \alpha \left(\frac{1}{2} \frac{dE^2}{dx} + 2E^2 \right), \quad (7)$$

where $E \equiv H/H_0$, $\Omega_k = -k/H_0^2$ and $\Omega_{m0} = \frac{8\pi G \rho_{m0}}{3H_0^2}$ are adopted. Solving this equation, one has

$$\begin{aligned} E^2 &= \Omega_k e^{-2x} + \Omega_{m0} e^{-3x} + \frac{\alpha}{2 - \alpha} \Omega_{m0} e^{-3x} + f_0 e^{-(4 - \frac{2}{\alpha})x} \\ &= \Omega_k e^{-2x} + \Omega_{m0} e^{-3x} + \Omega_R(x), \\ &= \Omega_k (1 + z)^2 + \Omega_{m0} (1 + z)^3 + \Omega_R(z) \end{aligned} \quad (8)$$

where f_0 is the integral constant which is determined by the current values of cosmological parameters, Ω_R is defined as the dimensionless energy density of RDE in terms of x or redshift z

$$\begin{aligned} \Omega_R(x) &= \frac{\alpha}{2 - \alpha} \Omega_{m0} e^{-3x} + f_0 e^{-(4 - \frac{2}{\alpha})x} \\ &= \frac{\alpha}{2 - \alpha} \Omega_{m0} (1 + z)^3 + (1 - \Omega_k - \frac{2}{2 - \alpha} \Omega_{m0}) (1 + z)^{(4 - \frac{2}{\alpha})}. \end{aligned} \quad (9)$$

In the second equal sign of the above equation, the initial condition $E_0 = 1$, i.e. $\Omega_k + \Omega_{m0} + \Omega_{R0} = 1$ is used. Its equivalent $f_0 = 1 - \Omega_k - \frac{2}{2 - \alpha} \Omega_{m0}$ is given.

Considering the conservation equation of RDE, one has

$$w_R = -1 - \frac{1}{3} \frac{d \ln \Omega_R}{dx} = -1 + \frac{(1 + z)}{3} \frac{d \ln \Omega_R}{dz}. \quad (10)$$

Now, we give a discussion on RDE model. In the above review, the case of $\alpha = 2$ is not taken into accounts. When $\alpha = 2$, one has the solution of Eq. (8) in terms of x and redshift z as follows

$$\begin{aligned} E^2 &= f_0 e^{-3x} + \Omega_k e^{-2x} - \Omega_{m0} x e^{-3x} \\ &= f_0 (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_{m0} (1 + z)^3 \ln(1 + z) \\ &= \Omega_{m0} (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_R(z), \end{aligned} \quad (11)$$

where f_0 is an integral constant. In the third equal sign of above equation, noticing the first term behaves like cold dark matter, we let $f_0 = \Omega_{m0}$ and defined $\Omega_R(z) = \Omega_{m0} (1 + z)^3 \ln(1 + z)$ as RDE. While considering the current values of cosmological parameters, one finds that $\Omega_{R0} = 0$ which is not consistent with the cosmic observations. Then, the case of $\alpha = 2$ is not a viable dark energy model. Once one has $\alpha \neq 2$, it can be seen from Eq. (9) that the RDE contains two terms: the first one behaves like cold dark matter, and the second one behaves like exotic energy component which properties are determined by the parameter α . For its importance of the parameter α , in this paper, we will confront this model with current cosmic observations including type Ia supernovae (SN Ia), baryon acoustic oscillations (BAO) and cosmic microwave background (CMB) to test its viability.

The maximum likelihood method will be used to constrain the RDE model. The SN Ia data used in this paper contains 192 SN Ia data [21] compiled from the ESSENCE [22] and Gold sets [23, 24, 25]. Constraints from Sne Ia can be obtained by fitting the distance modulus $\mu(z)$

$$\mu_{th}(z) = 5 \log_{10}(D_L(z)) + \mathcal{M}, \quad (12)$$

where, $D_L(z)$ is the Hubble free luminosity distance $H_0 d_L(z)/c$ and

$$d_L(z) = c(1+z) \int_0^z \frac{dz'}{H(z')} \quad (13)$$

$$\begin{aligned} \mathcal{M} &= M + 5 \log_{10} \left(\frac{cH_0^{-1}}{\text{Mpc}} \right) + 25 \\ &= M - 5 \log_{10} h + 42.38, \end{aligned} \quad (14)$$

where, M is the absolute magnitude of the object (SN Ia here), and H_0 is the Hubble constant which is denoted in a renormalized quantity h defined as $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$. For SN Ia dataset, the best fit values of parameters in a model can be determined by minimizing

$$\chi_{SN Ia}^2(p_s) = \sum_{i=1}^N \frac{(\mu_{obs}(z_i) - \mu_{th}(p_s; z_i))^2}{\sigma_i^2}, \quad (15)$$

where $N = 192$ for the combined SN Ia dataset, $\mu_{obs}(z_i)$ is the distance moduli obtained from observations, σ_i is the total uncertainty of the SN Ia data, and p_s denotes the parameters contained in the model.

The size of Baryon Acoustic Oscillation (BAO) is found by Eisenstein *et al* [26] by using a large spectroscopic sample of luminous red galaxy from SDSS and obtained a parameter A which does not depend on dark energy models, in a flat universe. And, it is written as

$$A = \frac{\sqrt{\Omega_{m0}}}{E(z_{bao})^{1/3}} \left[\frac{1}{z_{bao}} \int_0^{z_{bao}} \frac{dz}{E(z)} \right]^{2/3}, \quad (16)$$

where, $z_{bao} = 0.35$ and $A = 0.469 \pm 0.017$. The parameter has been used to constrain the dark energy models. From BAO, the best fit values of parameters in dark energy models can be determined by minimizing

$$\chi_{BAO}^2(p_s) = \frac{(A(p_s) - 0.469)^2}{0.017^2}. \quad (17)$$

The constraint to dark energy from CMB can be used is the CMB shift parameter R [27],

$$R = \sqrt{\Omega_{m0}} \int_0^{z_{rec}} \frac{dz}{E(z)}, \quad (18)$$

where $z_{rec} = 1089$ is the redshift at the last scattering surface. The R obtained from 3-year WMAP data [4] is [28]

$$R = 1.70 \pm 0.03. \quad (19)$$

From CMB constraint, the best fit values of parameters in dark energy models can be determined by minimizing

$$\chi_{CMB}^2(p_s) = \frac{(R(p_s) - 1.70)^2}{0.03^2}. \quad (20)$$

For Gaussian distributed measurements, the likelihood function $L \propto e^{-\chi^2/2}$, where χ^2 is

$$\chi^2 = \chi_{SN Ia}^2 + \chi_{BAO}^2 + \chi_{CMB}^2, \quad (21)$$

here $\chi_{SN Ia}^2$ is given in Eq. (15), χ_{BAO}^2 is given in Eq. (17), χ_{CMB}^2 is given in Eq. (20). Following the process described above, the maximum likelihood corresponds to the minimum of χ^2 . We list the calculation results in Tab. I. Also, the evolutions of EOS of RDE and the contour with 1σ and 2σ error regions are plotted in Fig. 1.

From Tab. I. and Fig. 1, one can find that the EoS of RDE is less than -1 at present with 1σ error. In the past, i.e. at high redshift, the EoS approaches zero. So, RDE behaves like cold dark matter in the past and like phantom at present. So, it is a quintom like dark energy model.

In summary, in this paper, the viability of Ricci dark energy is tested by confronting it with current cosmic observations including type Ia supernovae (SN Ia), baryon acoustic oscillations (BAO) and cosmic microwave background (CMB). By the maximum likelihood method, we find the minimum $\chi_{min}^2 = 202.25$ and the best fit parameters: $\Omega_{m0} = 0.34 \pm 0.04$, $\alpha = 0.38 \pm 0.03$ with 1σ error. With these best fit values of the parameters, one found out that Ricci dark energy is a viable dark energy model.

Datasets	χ^2_{min}	Ω_{m0}	α
SN+BAO+CMB	202.25	0.34 ± 0.04	0.38 ± 0.03

TABLE I: The values of minimum χ^2 and best fit values of the parameters.

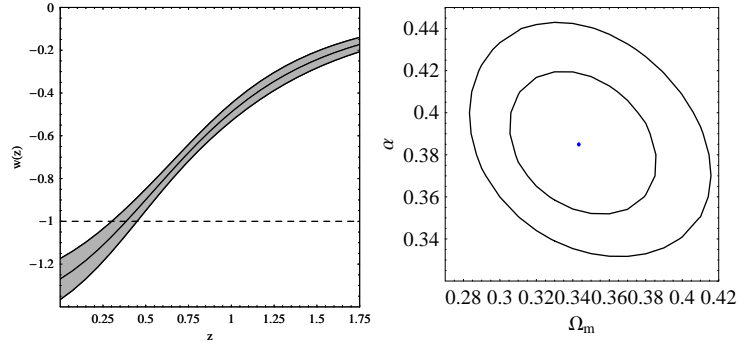


FIG. 1: The evolutions of EoS of RDE (the left panel) with respect to the redshift z and contour (the right panel) of Ω_{m0} and α , where the parameters Ω_{m0} and α are determined from SN+BAO+CMB. The center solid line is plotted with the best fit values, where the shadows denote the 1σ regions.

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